 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2010

# MT 1810/ 1804 – LINEAR ALGEBRA

Date : 30-10-2011 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

1. a) (i) Prove that similar matrices have the same characteristic polynomial.

(OR) (5)

(ii) Let T be the linear operator on ℜ3 which is represented in the standard ordered basis by

the matrix . Find the characteristic polynomial of A.

b) (i) State and prove Cayley-Hamilton theorem.

(OR) (15)

(ii) Let V be a finite dimensional vector space over F and T a linear operator on V. Then

prove that T is diagonalizable if and only if the minimal polynomial for T has the

form are distinct elements of F.

II. a) (i) Let T be a linear operator on a finite dimensional space V and let c be a scalar. Prove

that the following statements are equivalent.

1. c is a characteristic value of T.
2. The operator (T − cI) is singular.
3. det (T − cI) = 0.

(OR) (5)

(ii) Let W be an invariant subspace for T. Then prove that the minimal polynomial for Tw

divides the minimal polynomial for T.

b) (i) State and prove Primary Decomposition theorem.

(OR) (15)

(ii) Let T be a linear operator on a finite dimensional space V. If T is diagonalizable and if

c1,…,ck  are the distinct characteristic values of T, then prove that there exist linear

operators E1,…,Ek on V such that

1. T = c1E1 +…+ ckEk.
2. I = Ej +…+ Ek.
3. EiEj = 0,i≠j.
4. Each Ei is a projection

III. a) (i) Let W be a proper T-admissible subspace of V. Prove that there exists a nonzero α in

V such that W ∩ Z (α ; T) = {0}.

(OR) (5)

(ii) Define T− annihilator, T--admissible, Projection of vector space V and Companion

matrix.

b) (i) State and prove Cyclic Decomposition theorem.

(OR) (15)

**(P.T.O.)**

ii) Let P be an m x m matrix with entries in the polynomial algebra F [x]. The following are

equivalent.

1. P is invertible
2. The determinant of P is a non-zero scalar polynomial.
3. P is row-equivalent to the m x m identity matrix.
4. P is a product of elementary matrices.

IV. a) (i) Let V be a complex vector space and *f* be a form on V such that f (α,α) is real for

every α. Then prove that f is Hermitian. (5)

(OR)

ii) Let *f ­* be the form on a finite-dimensional complex inner product space V. Then prove

that there is an orthonormal basis for V in which the matrix of *f* is upper-triangular.

b) (i) Let *f* be a form on a finite dimensional vector space V and let A be the matrix of *f* in an

ordered basis B. Then *f* is a positive form iff A = A\* and the principal minors of A are all

positive.

(OR) (15)

(ii) Let V be a finite-dimensional inner product space and f a form on V. Then show that there is a

unique linear operator T on V such that f(α,β) = (Tα⏐β) for all α, β in V, and the map f →T is an

isomorphism of the space of forms onto L(V,V).

V. a) (i) Let V be a vector space over the field F. Define a bilinear form *f* on V and prove that the

function defined by *f* *(α;β)* = L1 (α) L2 (β) is bilinear.

(OR) (5)

ii) Define the quadratic form q associated with a symmetric bilinear form f and prove that



b) i)Let V be a finite dimensional vector space over the field of complex numbers. Let f be a symmetric bilinear form on V which has rank r. Then prove that there is an ordered basis

B ={β1, β2, … βn} forV such that the matrix of f in the ordered basis B is diagonal and .

(OR) (15)

ii) If f is a non-zero skew-symmetric bilinear form on a finite dimensional vector space V then

prove that there exist a finite sequence of pairs of vectors, (α1, β1), (α2, β2),… (αk, βk) with the

following properties.

1. f (αj, βj) = 1 , j=1,2,,…,k.
2. f (αi, αj)=f(βi, βj­)=f(αi,βi)=0,i≠j.

c) If Wj is the two dimensional subspace spanned by αj and βj, then V=W1 ⊕ W2⊕ …Wk ⊕ W0

where W0 is orthogonal to all αj and βj and the restriction of *f*  to W0 is the zero form.

**$$$$$$$**